# Phase Shift by Periodic Loading of Waveguide and Its Application to Broad-band Circular Polarization

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*Summary*—A rectangular or square waveguide may be loaded periodically by thin capacitive or inductive irises in order to produce phase delay or phase advance, respectively. The amount of phase shift may be calculated with accuracy by making use of available theoretical values of iris susceptance and of transmission line theory. The phase shifting sections may be designed for low voltage standing-wave ratio (vswr) over a considerable bandwidth.

When a square waveguide capable of supporting two fundamental modes is loaded periodically, the irises act inductively for one mode and capacitively for the other, thus introducing a differential phase shift. This differential phase shift may be made equal to 90°, in order to convert linear to circular polarization. Furthermore such a device may be made, by proper choice of parameters, to yield near-circular polarization over a bandwidth of 1.65:1, because the variation in phase delay for one mode and phase advance for the other tend to compensate each other as the frequency is varied.

Several of these circular polarizers have been built and tested at $X$ band and the measured results of ellipticity and vswr, as well as broad-band performance check with theoretical values quite closely.

## Introduction

PERIODIC loading of a transmission line with lumped reactances is a well-known technique for varying the transmission characteristics of the line. Rectangular or square waveguide may be loaded with irises which, if far enough apart so that higher-mode coupling is negligible, act as lumped loading on a uniform transmission line. The irises, if very thin, may be represented by shunt capacitance or inductance. By use of available theoretical values of susceptance[1] and of transmission line theory, the phase delay or advance as well as the voltage standing-wave ratio (vswr) introduced by the irises may be readily calculated. By proper design, low vswr may be achieved as well as a phase shift, which proves on experimental verification to be very close to the calculated value.

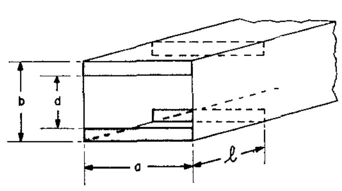


Fig. 1—Square waveguide loaded with irises.

When such irises are introduced into waveguide of square cross section (Fig. 1), they act as shunt capacitance for one mode ($TE_{01}$) and inductance for the perpendicular mode ($TE_{10}$). Thus one mode is delayed in phase while the other is advanced, and the total phase difference for the two modes may be made equal to 90°. Furthermore, since the effect of capacitive irises increases with increasing frequency while that of inductive irises decreases, this 90° phase difference between the two modes may be held nearly constant over a broad band of frequencies. Such a scheme is of course also useful where a phase difference approximately constant over a band is desired between two separate waveguide channels.

## Analysis

The analysis of the differential phase shift structure has been carried out by considering each mode in square waveguide as a uniformly loaded line which may be represented by an equivalent transmission line of new characteristic impedance and propagation constant[2] (Fig. 2). Considering one section of the line,

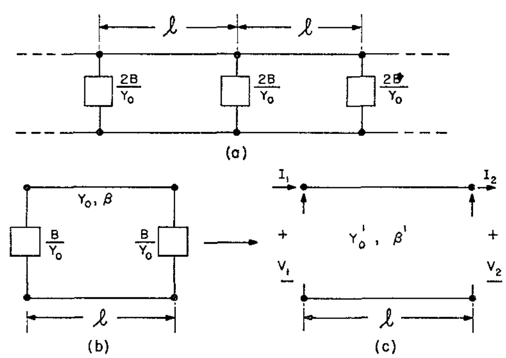


Fig. 2—Periodically loaded transmission line and its equivalent line; (a) loaded line, (b) one π-section of line, (c) one section of equivalent line.

$\beta$ is the propagation constant of the original line,
$\beta'$ is the propagation constant of the equivalent line,
$l$ is the length of the section,
$B/Y_0$ is the normalized susceptance of the iris,
$Y_0$ is the admittance of the original line,
$Y_0'$ is the admittance of the equivalent line; then

$$\beta' l = \cos^{-1}[\cos \beta l - B/Y_0 \sin \beta l] \tag{1}$$

$$\frac{Y_0'}{Y_0} = \left[1 - \frac{B^2}{Y_0^2} + 2\frac{B}{Y_0}\cot \beta l\right]^{1/2}. \tag{2}$$

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[1] Marcuvitz, N., "The Waveguide Handbook," M.I.T. Rad. Lab. Series, New York, McGraw-Hill, Chap. 10, pp. 218–224, 1951.

[2] These results are similar to those given by S. B. Cohn in "Analysis of the metal-strip delay structure for microwave lenses," *Jour. Appl. Phys.*, vol. 20, pp. 257–262; March, 1949.

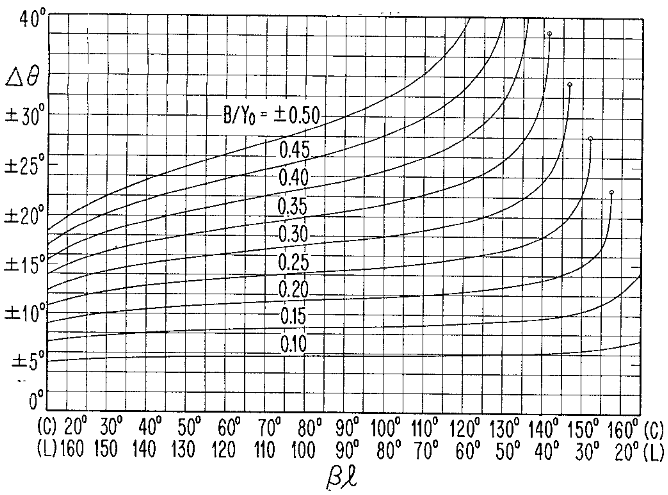


Fig. 3—Phase shift as a function of susceptance and iris spacing.

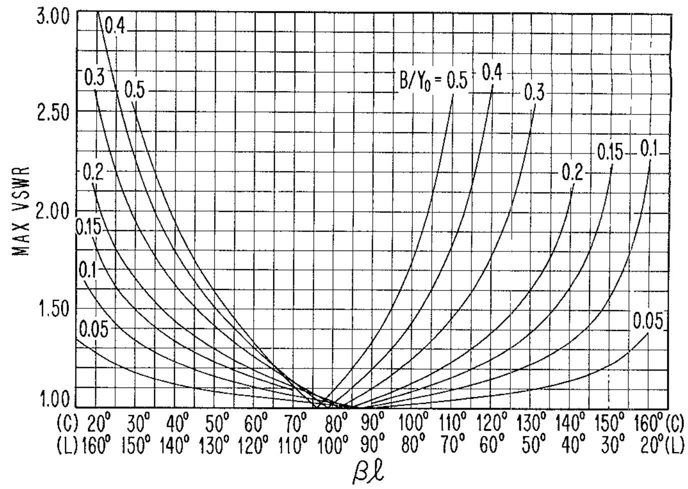


Fig. 4—Maximum possible vswr as a function of susceptance and iris spacing.

Phase shift that has been added per section is given by

$$\Delta\theta = \beta' l - \beta l, \tag{3}$$

which is plotted on Fig. 3 for some values of $B/Y_0$ and $\beta l$. Note that $\Delta\theta$ is positive for $B/Y_0 > 0$, (capacitive case), negative for $B/Y_0 < 0$ (inductive case), and that the same curves are used for capacitive and inductive irises with a change of scale on the abscissa. The normalized characteristic admittance may be used to calculate the vswr. If the electrical length of the total equivalent line is equal to an odd-multiple quarter wavelength, a maximum vswr is introduced and is equal to $(Y_0'/Y_0)^2$ or $(Y_0/Y_0')^2$, whichever is greater than 1. This maximum possible vswr is plotted in Fig. 4 and is seen to remain low over a band of frequencies if the values of $B/Y_0$ are small.

For each value of $B/Y_0$, a value of $\beta l$ can be found for which a perfect match is obtained. This occurs when

$$\cot \beta l = \frac{1}{2} \frac{B}{Y_0}, \tag{4}$$

and the phase shift for this matched phase shifter is

$$\Delta\theta = 180° - 2\beta l. \tag{5}$$

Thus, if a given phase shift per section is desired at a given frequency, $\beta l$ may be determined from (5) and $B/Y_0$ from (4). To illustrate the use of (4) and (5) in obtaining a simple phase shift in rectangular waveguide, the phase shifter pictured in Fig. 5 was constructed and tested. Table I gives pertinent data. This indicates the accuracy with which even such a large phase shift can be predicted.

TABLE I

| Frequency | $\beta l$ | $B/Y_0$ | Design Phase Shift | Measured Phase Shift | Measured vswr |
|---|---|---|---|---|---|
| 9375 mc | 64.5° | 0.952 | 102° | 111°±3° | 1.06 |

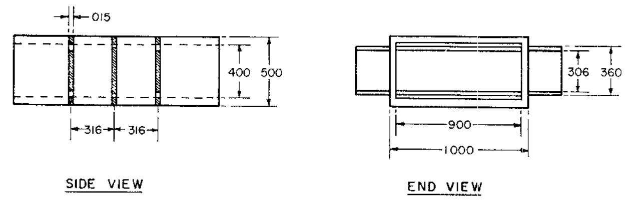


Fig. 5—Phase shifter in rectangular waveguide.

Of major interest was the application of this phase shift theory to producing differential phase shift in square waveguide to obtain circular polarization over a broad band. For a study of the broad-band characteristics of such a differential phase shifter, the variation of phase shift and match with frequency may be obtained by use of (1), (2), and (3). Of course the value of $B/Y_0$ varies with frequency, and this also must be considered. It can be shown theoretically that for capacitive irises

$$\frac{B_c}{Y_0} = K_C \frac{b}{\lambda_g},$$

and for inductive irises,

$$\frac{B_L}{Y_0} = -\frac{K_L}{2} \frac{\lambda_g}{b},$$

where

$b$ is the guide height,
$\lambda_g$ is the guide wavelength,
$K_C$ and $K_L$ are functions of the iris geometry which vary slowly with frequency and are approximately equal for small values of susceptance. (See Fig. 6.)

Using the above theory, differential phase shift and vswr were studied as a function of all the parameters of the waveguide geometry (see Fig. 1) and of frequency in an attempt to determine those values which gave most nearly constant phase difference and low vswr over the broadest frequency bands. The results of this theoretical study were as follows:

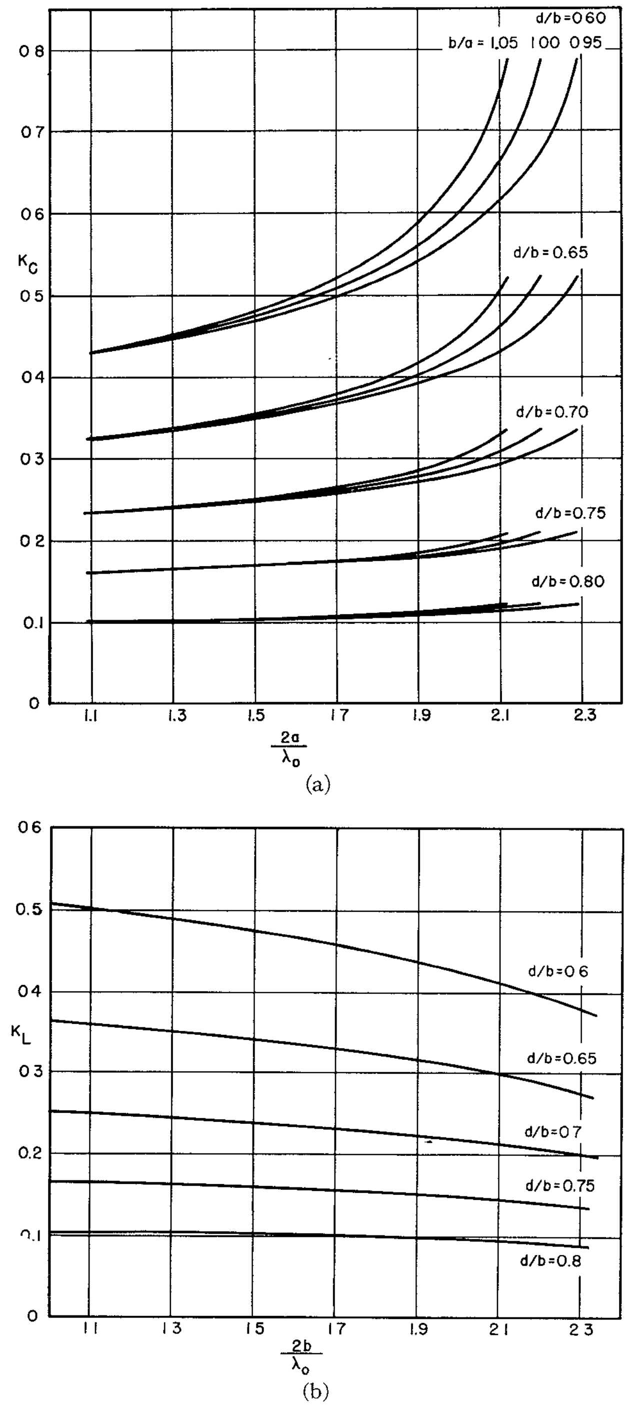


Fig. 6(a,b)—$K_C$ and $K_L$ as functions of iris geometry and free-space wavelength, $\lambda_0$.

1. Optimum bandwidth is achieved with a square cross section ($b/a=1$). Varying $b/a$ slightly about this point shifts the center frequency of the band and narrows the bandwidth.

2. The bandwidth is optimized for spacing between irises of approximately $l=0.4b$, though this parameter is not critical.

3. Optimum bandwidth (as well as lowest vswr) is achieved with smallest iris susceptance. Practical considerations limit the improvement to be obtained in this direction, because smaller susceptance means less phase shift per section and thus a longer structure. The longer structure makes tolerances on squareness of waveguide more critical. In addition, if the iris fins become very small, the assumption of infinitesimal iris thickness is no longer very good, since the ratio of thickness to height must be very small for this approximation to be satisfactory. In any case, the bandwidth is limited theoretically to be less than 2:1, for the periodically loaded structure acts as a band-pass filter.

4. Broad-band behavior exists in a range of frequencies from about 1.30 $f_c$ to 2.1 $f_c$, where $f_c$ is the cutoff frequency for the lowest ($TE_{10}$) mode. Higher modes can exist in the waveguide over this range, but experimental results indicated no difficulty on this score, probably because of symmetry of feeding structure.

## Experimental Results

Several 90° differential phase shifters were built according to the preceding theory and used to convert a linearly polarized mode into a circularly polarized mode in square waveguide. Measurement of axial ratio of the polarization of the wave radiated from the open end of the waveguide was used as an indication of the phase difference. The sections containing the irises were fed from a transition from rectangular to square waveguide with the incident linear polarization at 45° to the sides of the square waveguide (see Fig. 7). The results ob-

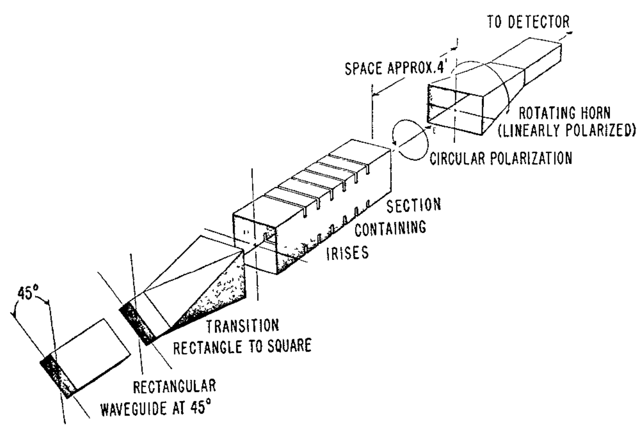


Fig. 7—Test setup, schematic view.

tained agreed very closely with those predicted by the theory, except in the case of very small fins, where the neglect of the finite iris thickness is believed to be at fault. Phase shift approximately 10 per cent higher than predicted was obtained in this case (see Fig. 8(b)). Results for three designs are shown in Fig. 8. Fig. 9 shows a typical design. Measurements on this circular polarizer, indicated in Fig. 8(a), showed that the ellipticity of polarization remained below 3 db from 7,350 to 10,300 mc and the vswr for either polarization in the square waveguide was 1.5 or less.

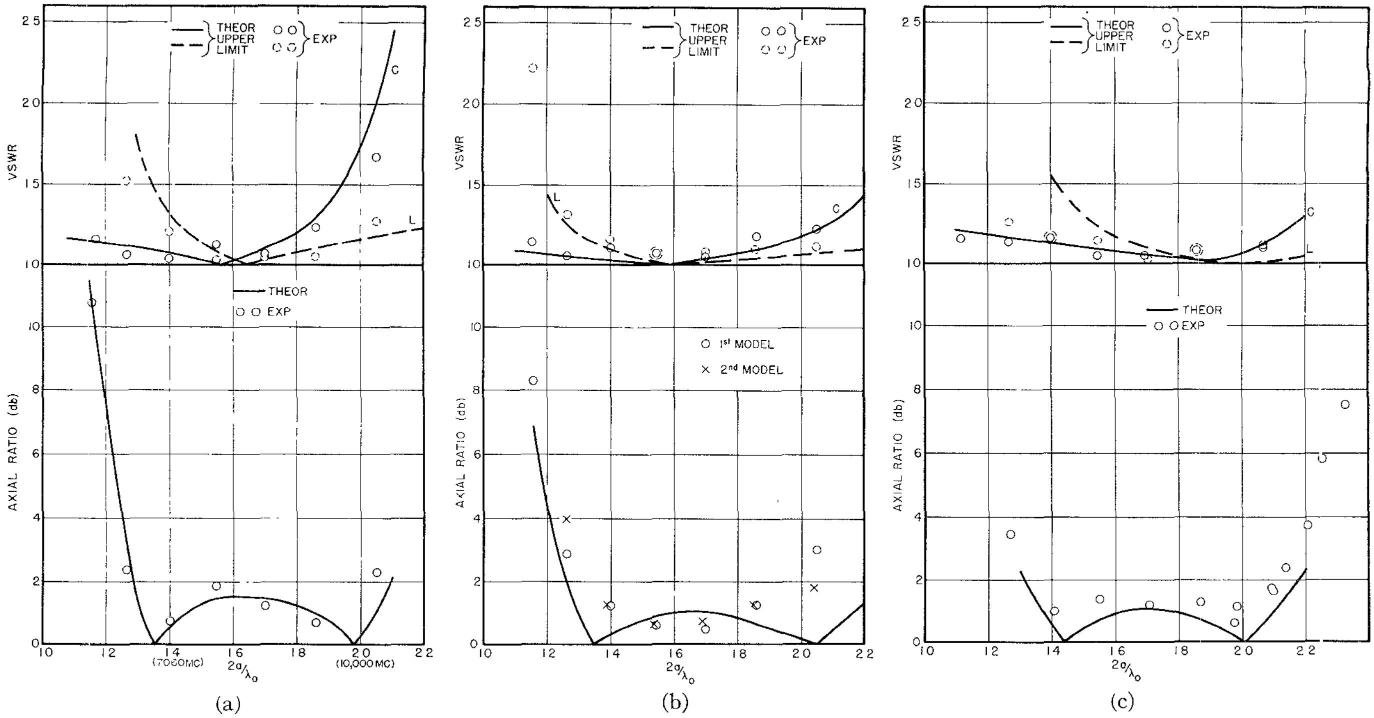


Fig. 8—Measured axial ratio and vswr; (a) $l/b=0.4$, $d/b=0.7$, $b/a=1.00$ (5 irises), (b) $l/b=0.4$, $d/b=0.8$, $b/a=1.00$ (11 irises), (c) $l/b=0.3$, $d/b=0.726$, $b/a=1.00$ (6 irises).

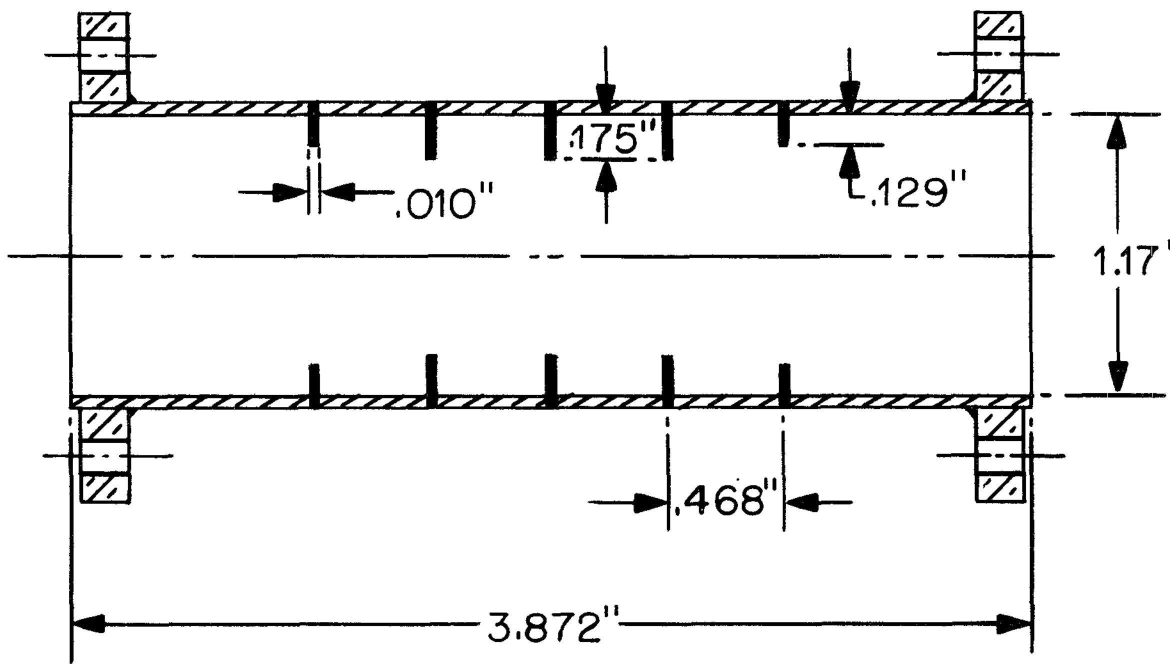


Fig. 9—Five-iris circular polarizer.